

# A Family of Distribution-Entropy MAP Speckle Filters for Polarimetric SAR Data, and for Single or Multi-Channel Detected and Complex SAR Images

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## ABSTRACT

Five new Distribution-Entropy Maximum A Posteriori (DE MAP) speckle filters are established for the following cases: single detected, multilook multi-channel detected, single look complex SAR images, separate complex looks, and fully polarimetric SAR data.

As shown, these new filters are particularly efficient to reduce speckle noise, while preserving textural properties and spatial resolution, especially in strongly textured SAR images.

Keywords: Speckle filtering, Maximum Entropy, Maximum A Posteriori filters, Complex SAR images, multi-temporal SAR data, multi-channel SAR data, Polarimetric SAR images.

## 1. DE-MAP SPECKLE FILTERING

In the case of multi-channel detected SAR images, let define the vector quantities of interest:  $I$  is the speckled intensity vector available in the actual SAR data;  $R$  is the radar reflectivity vector which is the quantity we want to restore. The Maximum A Posteriori (MAP) filtering method bases on the famous Bayes' theorem:

$$P(R/I) = P(I/R) \cdot P(R) / P(I) \quad (1)$$

For multi-channel detected SAR images, MAP Filtering is a vector filtering method. For any channel  $i$ , the posterior probability is maximum if the following condition is verified:

$$\frac{\partial \ln(P(I/R))}{\partial R_i} + \frac{\partial \ln(P(R))}{\partial R_i} = 0 \text{ for } R_i = R_{i\_MAP} \quad (2)$$

In presence of very strong texture, as it is often the case in SAR images of dense tropical forest, and in the presence of relief, it may be hazardous to make an

assumption about the probability density function of the radar reflectivity.

In this context, the A Priori knowledge with regard to the observed scene can hardly be an analytical first order statistical model. However, we introduce a Maximum Entropy (ME) constraint on texture [1-3]:

$$S(R_i) = - \sum_k [R_{ik} \cdot \log(R_{ik})] \text{ for the } i^{\text{th}} \text{ channel} \quad (3)$$

Because the  $R_k$  are non-negative and  $\exp(S(R_i))/Z$  is normalized, it can be treated as a probability density function (pdf) whose entropy is  $S(R_i)$  [1]:

$$P(R_i) = 1/Z \cdot \exp(S(R_i)) = 1/Z \cdot \exp(-\sum_k [R_{ik} \cdot \log(R_{ik})]) \quad (4)$$

To estimate  $P(R_i)$ , the radar reflectivities  $R_{ik}$  are estimated by deconvolution from the SAR impulse response function as described in Nezry *et al.*, 1995 [4].

## 2. DE-MAP FILTERS FOR DETECTED SAR IMAGES

### Single detected SAR image case:

For a single detected SAR image, the conditional pdf of the speckle can be modelled as a Gamma distribution:

$$P(I/R) = (L/R)^L / \Gamma(L) \cdot \exp(-LI/R) \cdot I^{L-1} \quad (5)$$

With this assumption, the *Gamma/Distribution-Entropy MAP (Gm-DE MAP) filter for single-channel detected SAR images* is the solution of the following equation:

$$L I - LR - R^2 \cdot \sum_k [\log(R_k) - 1/\ln(10)] = 0 \quad (6)$$

Figure 1 and 2 illustrate the application of this speckle filter to a ERS-PRI (3-looks) SAR image. Figure 1 is the original ERS-PRI image, and Figure 2 is its filtered version.

**Detected multi-channel SAR data case:**

For multilook SAR images, the conditional pdf of the speckle  $P(I/R)$  can be modelled as a multivariate Gaussian distribution [5], in the case the speckle is correlated between image channels:

$$P(I/R) = [(2\pi)^N |\text{Cov}_s|]^{-1/2} \cdot \exp[-{}^t(I-R) \cdot \text{Cov}_s^{-1} \cdot (I-R)] \quad (7)$$

The ME constraint on texture is introduced as above. With this technique, there is no need to introduce explicit correlation between the diverse texture channels.

Under these assumptions, the *Gaussian/Distribution-Entropy MAP (Gs-DE MAP) filter for multi-channel multilook detected SAR images* (N channels) comes down to the resolution of a set of N coupled scalar equations of the form:

$${}^t(1_i) \cdot C_s^{-1} \cdot (I-R) + {}^t(I-R) \cdot C_s^{-1} \cdot (1_i) - R_i^2 \cdot \sum_k [\log(R_{ik}) - 1/\text{Ln}(10)] = 0 \quad (8)$$

Note that in the case the speckle is not correlated between image channels, the *Gamma/Distribution-Entropy MAP (Gm-DE MAP) filter for multi-channel detected SAR images* (N channels) comes down to the resolution of a set of N independent (uncoupled) scalar equations similar to Equation (6).

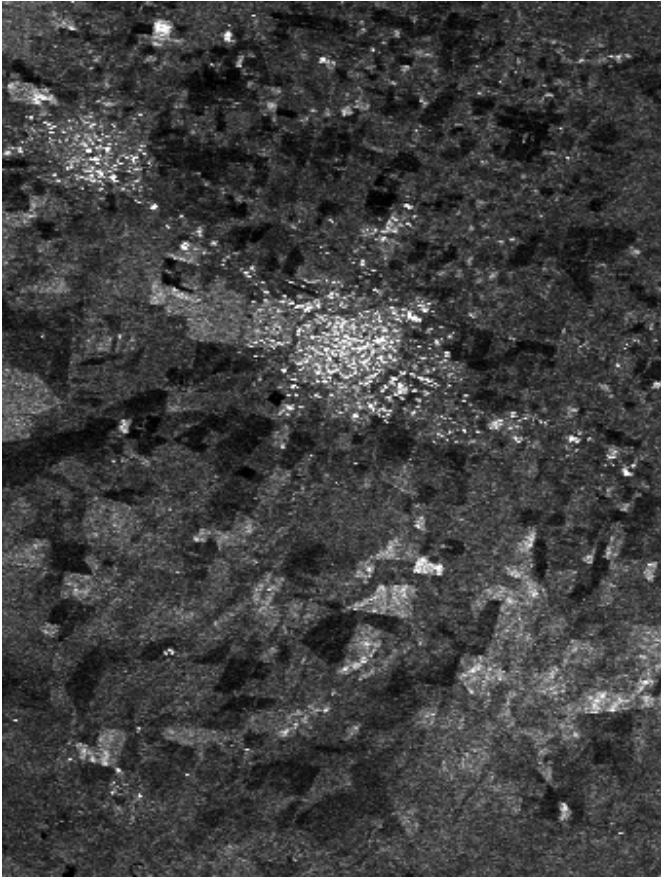


Figure 1: Unfiltered ERS PRI image.

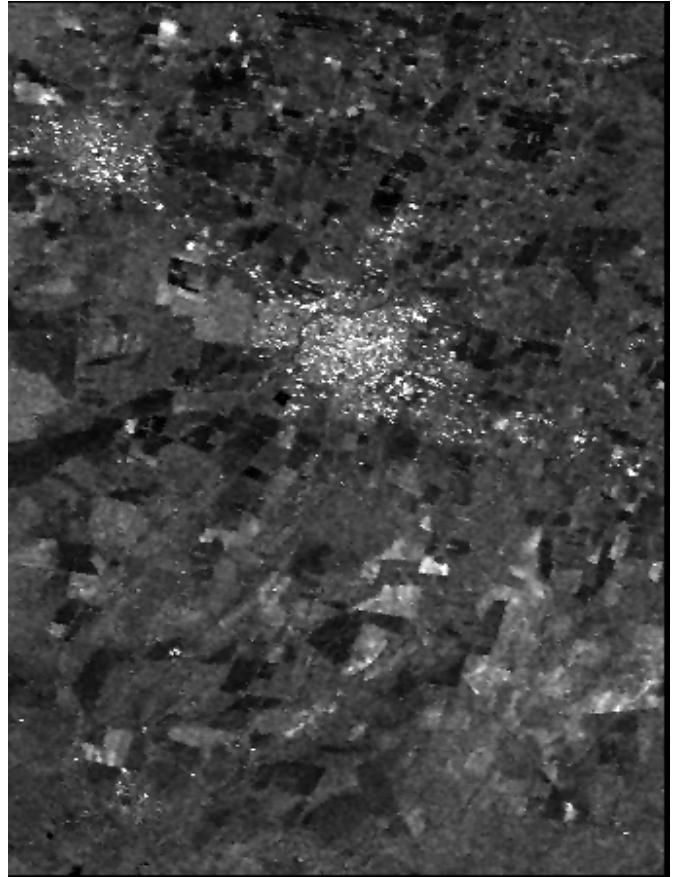


Figure 2: DE-MAP filtered version of the ERS SAR image.

### 3. DE-MAP FILTERS FOR COMPLEX SAR DATA

The  $L$  separate complex looks are usually extracted from the useful Doppler band, at the cost of the azimuth resolution. From the point of view of the speckle-filtering-problem, the case of interferometric complex SAR data sets can be considered in a similar way. The measurement vector for each pixel is  $X=\{y_n\}$ , where  $y_n=i_n+jq_n$ .

When speckle is fully developed, the  $(i_n, q_n)$  are statistically independent random processes. However, the  $y_n$  are correlated complex Gaussian random processes with pdf given by Goodman, 1985 [6]:

$$P(X/C_S) = \exp(X^* \cdot C_S^{-1} \cdot X) / (\pi^L |C_S|) \quad (9)$$

#### Separate complex looks case:

In this case, the *Complex-Gaussian/Distribution-Entropy MAP (CGs-DE MAP) filter for separate complex looks* ( $L$  complex looks) is expressed as:

$$X^* \cdot C_S^{-1} \cdot X - LR_i - R_i^2 \cdot \sum_k [\log(R_{ik}) - 1/Ln(10)] = 0 \quad (10)$$

where  $C_S$  is the covariance matrix of the speckle between the complex separate looks [7].

In the case of a series of single-look complex (SLC) images (*e.g.* interferometric complex SAR data sets),  $C_S$  is the covariance matrix of the speckle between the SLC images, and the same filter can be applied, without loss in the spatial resolution.

#### Single-look complex (SLC) image case:

In this case, the *Complex-Gaussian/Distribution-Entropy MAP (CGs-DE MAP) filter for SLC SAR images* is expressed as follows:

$$1/N \cdot X^* \cdot C_S^{-1} \cdot X - R - R^2 \cdot \sum_k [\log(R_k) - 1/Ln(10)] = 0 \quad (11)$$

where  $C_S$  is the spatial covariance matrix of the complex speckle, and  $N$  is the number of pixels within the processing window.

### 4. DE-MAP FOR POLARIMETRIC SAR DATA

In the case of polarimetric SAR data,  $\Sigma_s$  is the polarimetric covariance matrix, and  $C_s$  is the unspeckled covariance matrix, *i.e.* the quantity to be restored through speckle filtering.

In the reciprocal case, and for low look correlation, the conditional pdf of  $\Sigma_s$  is a complex Wishart distribution of the form [8]:

$$P(\Sigma_s/C_s) = \frac{(\det \Sigma_s)^{L-3} L^{L-3}}{\pi^3 \Gamma(L) \Gamma(L-1) \Gamma(L-2) (\det C_s)^L} \exp[-\text{Tr}(L C_s^{-1} \Sigma_s)] \quad (12)$$

Using physical backscattering models, assuming (as a rough approximation) that texture is identical in all polarizations, we get the following approximation [8]:

$$C_s = \mu E(C_s) \quad (13)$$

where  $\mu$  is the scalar textural parameter equal to the normalized number of scatterers within the resolution cell, and  $E(C_s)$  is the mean covariance matrix [9].

With this assumption, the ME constraint on texture [1-3] becomes:

$$P(\mu) = 1/\mu \cdot \exp(-\sum_k [\mu_k \cdot \log(\mu_k)]) \quad \text{and} \quad E(\mu) = 1 \quad (14)$$

In this case, the *Complex-Wishart/Distribution-Entropy MAP (CW-DE MAP) filter for polarimetric multilook SAR data* is expressed as:

$$L \text{Tr}[E(C_s)^{-1} \Sigma_s] - L\mu - \mu^2 \cdot \sum_k [\log(\mu_k) - 1/Ln(10)] = 0 \quad (15)$$

$E(C_s)$  is obtained using the maximum likelihood estimator described in Lopès *et al.*, 1992 [8].

Figures 3 to 10 illustrate the application of this filter to high resolution P-band 4-look JPL AIRSAR polarimetric data. The restoration of the radar reflectivity is illustrated on Figures 3 and 4. The restoration of the degrees of coherence is illustrated on Figures 5 and 6; the restoration of the phase differences is illustrated on Figures 7 and 8. Figures 9 and 10 allow to appreciate how the CW-DE MAP filter and enhances the P-band polarimetric texture signatures [10] on the textured (forest) area indicated on Figures 3 and 4 (2035 pixels).

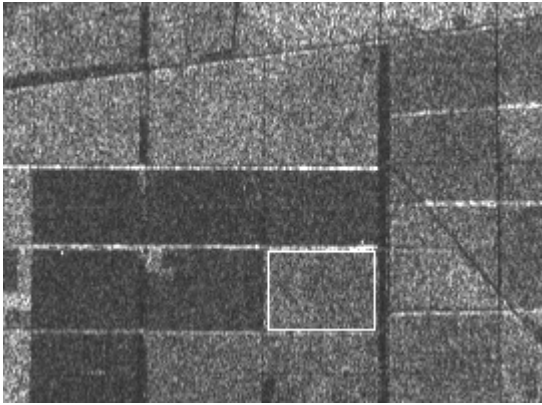


Figure 3: Original 4-look JPL/AIRSAR P-HH image.

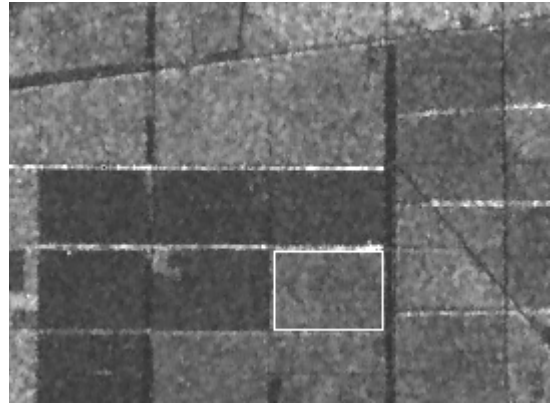


Figure 4: P-HH Pol. after CW-DE MAP filtering.

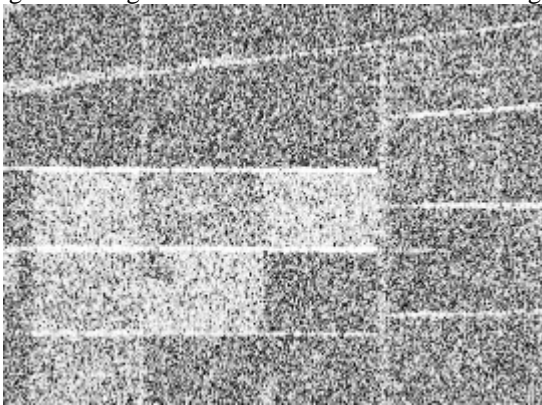


Figure 5: Unfiltered P-HH/VV degree of coherence.

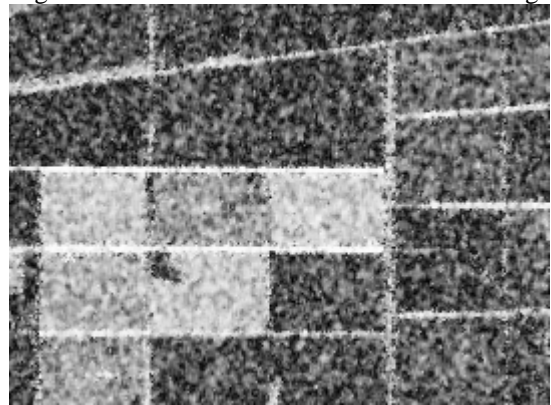


Figure 6: CW-DE MAP filtered degree of coherence.

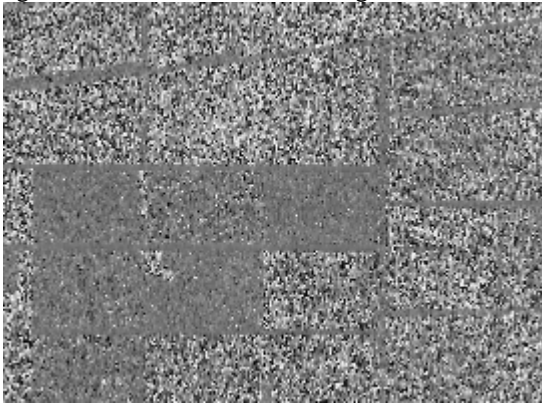


Figure 7: Unfiltered P-HH/VV phase difference

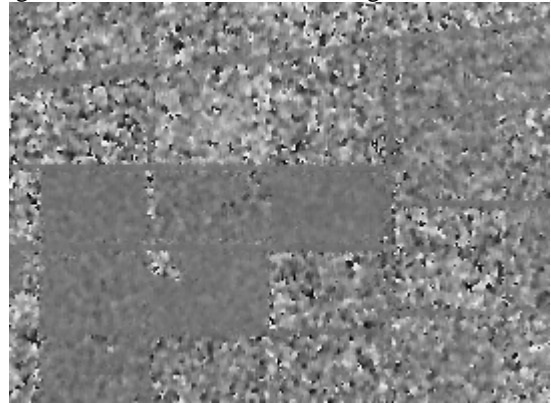


Figure 8: CW-DE MAP filtered HH/VV phase difference.

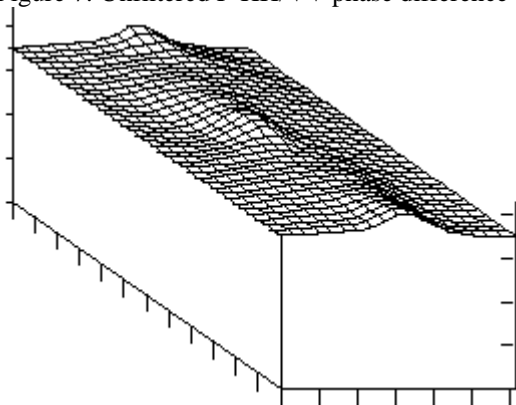


Figure 9: Original polarimetric texture signature.

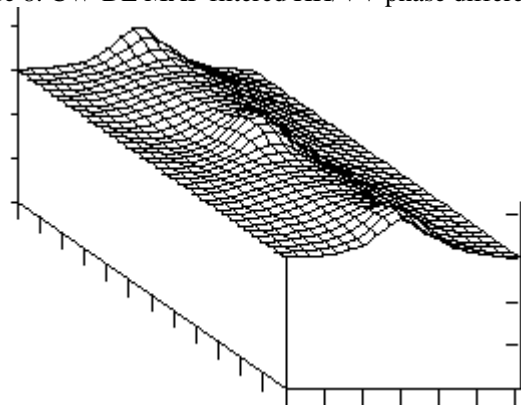


Figure 10: Texture signature after CW-DE MAP filtering.

## 5. CONCLUSION

The new DE-MAP filters presented above adapt to a much larger range of textures than the previous MAP filters [7,8,11,12] developed under the assumption of K-distributed SAR intensity. In particular, these filters might be of interest in the case of very high resolution SAR images.

The filtered images shown in Figure 2 and 4 show that performances in terms of speckle reduction, texture restoration (*cf.* [13]), as well as structures and point targets preservation are fully satisfactory. These filters have already proven a remarkable efficiency in operational remote sensing (*cf.* [14,15]).

From the theoretical point of view, it is noteworthy that these filters present the very interesting properties of control systems [12,16].

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